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Spice Simulation of High-Q Crystal Oscillators: Single and Dual-Mode Oscillator Analysis

A. A. Gubarev, A. V. Kosykh, S. A. Zavjalov, A. N. Lepetaev
Omsk State Technical University, Omsk, Russia
E-mail: oscillator@hotbox.ru

Abstract - **In the given paper the effective simulation technique of quartz oscillators in time domain with use of general purpose EDA software (PSPICE, MicroCAP, etc) is offered. Main idea is reception of averaged on the first harmonic parameters of nonlinear active part of self-oscillating circuit with the help of widespread programs of circuit simulation, with the subsequent analysis of complete self-oscillatory circuit by analytical methods. Detailed procedures of analysis of single and dual-mode crystal oscillators with use of MicroCAP 6 program are given.**

I. Introduction

The initial data for simulation of crystal oscillators (XO) are the oscillator circuit and model parameters of its components. Traditionally for analysis of XO properties both numerical, and analytical methods the complete model of XO uses. The complete XO model is obtain by direct connection of components models in a common equations system according to circuit topology. The nonlinear analysis of such model in time domain is associated with significant difficulties:

- The complexity of analytical representation of XO complete model restrict the usability of analytical methods only for simple circuits, although analytical approach allow to reduce of total amount of required calculations considerably. Besides, carrying out of analytical analysis demands the high qualification of researcher.
- General purpose EDA software (SPICE, Microcap), based on standard methods of numerical integration the ordinary differential equations (ODE) in time domain, are not effective for this task, owing to the big expenses of required calculation time. Because of high quality factor (Q) of resonator the analysis of transient demands a calculation of tens thousand of oscillation periods. There are special methods of numerical integration of multiperiod systems [1], accelerating the calculations, however the corresponding software is difficult to obtaining.

General purposes EDA software are the most preferable environment for XO simulation because of prevalence, availability and powerful computing and graphic opportunities. Therefore the problem of searching of new and more effective ways of XO simulation is topical. In some extent, this question is mentioned in works [2-6,8,9].

II. Methodology

The offered simulation technique is based on a use of macromodeling principles. The macromodel approximates complete model of a part of the circuit. It does not reflect an internal structure of the circuit any more, but represents the set of the relations, connecting only the input and the output state variables. Replacement of complete models to their macromodels allows transition to higher (macromodeling) level of modeling, which allows reducing dimension of solved tasks. Thus at different levels of modeling different methods of the analysis can be used. This can increase the general efficiency of modeling. The crystal oscillator can be described not only at a circuit level, but also on macromodeling level too.

In a general view, any crystal oscillator is considered as system of resonator and exciting circuit (osci – llator concept) [7]. There are two models of such system, described the occurring in oscillator processes - the model with a positive feedback and the model with negative resistance. Actually, models differ by representation of resonator exciting circuit: either as a two-port or one-port network (Fig. 1). Two-port representation of exciting circuit is described by averaged on first harmonic, complex Y, S etc parameters. One-port representation is described by averaged complex resistance Z.

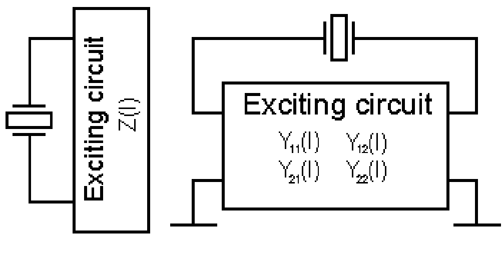


Fig.1 One-port and two-port representation of quartz oscillator.

The description of a nonlinear resonator exciting circuit by harmonically linearized parameters is possible because of the fact, that due to high Q of resonator, the current through a resonator within the oscillation period differs from harmonic negligible. Nonlinearity of exciting circuit is developed as dependence of its averaged on first harmonic parameters on the amplitude of oscillations.

TABLE 1
TWO-LEVEL SIMULATION OF CRYSTAL OSCILLATORS

	Simulation level	Simulation software	Input data	Output data	Mathematical tool	
					Models	Analysis methods
1	Circuit	General purpose EDA software (SPICE, MicroCAP)	Oscillator circuit	Exciting circuit macromodel parameters	Physical (SPICE) models of circuit components	Numerical methods of ODE solving
2	Macromodel (System)	Mathematical Software (MathCAD)	Resonator and exciting circuit macromodel parameters	Basic XO parameters (Self-excitation conditions, steady-state parameters, oscillations transient).	Model of "Resonator – Exciting circuit" system.	Approximate analytical methods of nonlinear oscillatory circuit analysis. Numerical methods of ODE solving

Representation of the crystal oscillator as Fig. 1 can be considered as its description at a macromodel level, where elementary units are macromodels of resonator and exciting circuit. Using of this level at the circuit analysis by only analytical or only numerical methods does not give any advantages. Therefore, traditionally the analysis is carried out at the circuit level with complete crystal oscillator model. However, the presence of two levels of simulation allows optimal combination of numerical and analytical analysis.

The basic advantage of combined numerical and analytical analysis is the significant reduction of simulation time in comparison to standard numerical analysis (200 times and more). Numerical methods effective dispatch with resonator less circuit analysis, and with it harmonically linearized macromodels parameters calculation. Parameters of excitation circuit macromodel can tell much about properties of crystal oscillator already in themselves. For example, having calculated the circuit parameters dependence on a resonator excitation current it is possible to estimate the conditions of the crystal oscillator excitation at different resonator activity. Besides, the calculations capacity required for transients calculation essentially reduces, since analytical methods allows reducing the ODE order.

Process of crystal oscillator simulation can be fully automated. At the macromodel level, crystal oscillator of any configuration, containing any passive and active elements, may be represented as the same circuit. It means that equations for "resonator – exciting circuit" system, obtained by analytical methods, can be applied to any crystal oscillator.

The submitted approach to oscillator simulation has a number of the restrictions connected to the following assumptions:

1. The resonator has high Q.
2. In comparison to resonator, other part of oscillator circuit is broadband (does not contain high-Q elements).
3. The oscillator circuit does not contain elements with large time constant, influencing on alternating current regime. Otherwise, it is necessary to increase the calculation time of exciting circuit macromodel parameters, because of a long transients setting. In some cases the calculations time increasing become unacceptable.

III. Single-mode crystal oscillator macromodel level circuit

The form of the resonator exciting circuit macromodel depends on:

1. Simulation task, i.e. on parameters of complete crystal oscillator model that must be investigated. For example, if it is necessary to study an influence of circuit component variation, the macromodel parameters should have a functional dependence on parameter of this component.
2. “Resonator - Exciting circuit” system model selection.

In our opinion, the most efficient is the model, where the exciting circuit represented by one-port, characterized by nonlinear complex impedance (negative resistance oscillator model).

In one-port representation, in comparison with two-port representation:

1. The procedure of macromodel parameters calculation becomes simpler.
2. Macromodel is characterized by only one nonlinear complex parameter Z instead of four Y or S parameters.
3. Calculation formulas obtained from the analytical analysis is easier and theirs form is more intuitive.

As essential disadvantage of this model, it is possible to consider the absence of a technique of resonator loaded Q calculation.

The using of one-port model allows considering crystal oscillator as following equivalent circuit (Fig. 2b).

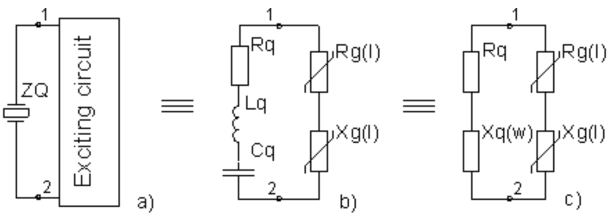


Fig. 2. Single-mode crystal oscillator macromodel level circuit.

Here Rq, Lq, Cq are parameters of the resonator equivalent circuit (resonator macromodel). The macromodel of exciting circuit is submitted by the one-port, by harmonically linearized nonlinear resistor. Thus the nonlinearity of the resistor is characterized by nonlinear functional dependence of it effective (averaged within a period of oscillations) resistance on amplitude of a resonator current. Rg(I), Xg(I) – are accordingly active and reactive components of this nonlinear resistance. Active component of the complex resistance is negative. The negative resistance compensates an energy loss in the resonator and sustains the oscillations. As a matter of convenience it is possible to use the equivalent submission fig. 2c, where Xq (w) - dependent on frequency resonator reactance.

In the circuit of Fig. 2 the following assumptions are used:

1. The resonator is described by a linear equivalent circuit as a sequential resonance circuit. Resonator static capacity is considered as an element of exciting circuit and taken into account in exciting circuit macromodel parameters.
2. Concerning the resonator, the exciting circuit is broadband. Therefore, resistance of the one-port, which replaces exciting circuit, in a range of possible frequency shifts is considered as constant (does not depend on frequency) [5].

The crystal oscillator analysis at macromodel level (circuit Fig. 2в) allows calculating the following oscillator parameters:

1. Self-excitation conditions. Are set by an inequality:
$$Rq + Rg(0) < 0\,, \tag{1}$$
where Rg(0) is active part of exciting circuit impedance at a zero resonator current. The self-excitation boundary is defined by maximum resistance of resonator, at which one the oscillator have self-excitation capability, and may be found from equality Rq+Rg(0)=0.

2. Steady-state parameters. Oscillations amplitude and frequency are defined by conditions of amplitudes and phases balance, which one for equivalent circuit of Fig. 2в looks like:
$$Rq + Rg(I) = 0 \tag{2}$$
$$Xq(w) + Xg(I) = 0 \tag{3}$$
The equation (2) determines the level of resonator current at steady state. On basis of (3), the oscillations frequency and frequency shift are calculated. The calculation of other parameters (voltages in circuit nodes, output power etc.) realize in simulation programs in transient analysis mode at resonator replacement by equivalent source of harmonic current with parameters are appropriate to steady state.

3. Amplitude and frequency transient. Transients are calculated by a solution of the nonlinear differential equation, which one for the equivalent circuit of Fig. 1в looks likes the following:
$$\frac{d^2 i}{dt^2} - w_q \cdot \frac{1}{Q} \cdot \left(\frac{-Rg(I)}{Rq} - 1 \right) \frac{di}{dt} + w_q^2 \cdot \left(\frac{Xg(I)}{Xq(w_q)} + 1 \right) i = 0\,, \tag{4}$$
where Q is resonator quality factor, $w_q^2 = \dfrac{1}{Lq \cdot Cq}$.

The shorted differential equations for oscillations amplitude and phase are obtained by the method of slowly varying amplitudes and written as:
$$\frac{dI}{dt} = \frac{I \cdot w_q}{2 \cdot Q} \cdot \left(\frac{-Rg(I)}{Rq} - 1 \right), \tag{5}$$
$$\frac{d\phi}{dt} = -\frac{w_q}{2} \cdot \frac{Xg(I)}{Xq(w_q)} = -\frac{w_q \cdot Xg(I)}{2 \cdot Q \cdot Rq}. \tag{6}$$
The solutions of equations (5) and (6) can be obtained with the help of any mathematical package, for example MathCAD. The equation (5) defines oscillation amplitude transient (in resonator), and equation (6) defines oscillation frequency transient.

4. Parametric sensitivity. For sensitivity calculation it is necessary to find functional dependence of exciting circuit macromodel parameters on parameters of elements of it complete (circuit) model.

This list can be continued for other tasks.

IV. Dual-mode Crystal Oscillator Macromodel Level Circuit

It is possible to apply approximations, used at the analysis of single-mode crystal oscillator for dual-mode crystal oscillator analysis. Since the quality-factor of resonator modes is great enough, the current through the resonator on each mode is practically harmonic (within one period of oscillations) Therefore, resonator exciting circuit can be considered as one-port under effect of two harmonic signals. On each of these signals the nonlinear one-port can be described by the harmonically linearized parameters. In this case, however, it is necessary to take into account, that the oscillations of both modes interact on nonlinearity of the one-port, and they cannot be considered separately. The interaction of modes in the exciting circuit can be taken into account if to present harmonically linearized parameters of the one-port on each oscillation mode as functions of two variables: functions of both oscillations amplitude. In dual-mode operation the one-port representation of resonator exciting circuit will be described by dependences:
$$Rg_k(I_1, I_2)\,,\; Xg_k(I_1, I_2)\,,\; k=1..2,$$
where k is number of oscillations mode.

If interaction between modes in resonator will not be taken into account (it is allowable at small levels of excitation and when frequency of one mode does not get in a band of other) and each oscillations mode will be replaced by linear equivalent circuit, dual-mode crystal oscillator can be described as equivalent circuit Fig. 3.

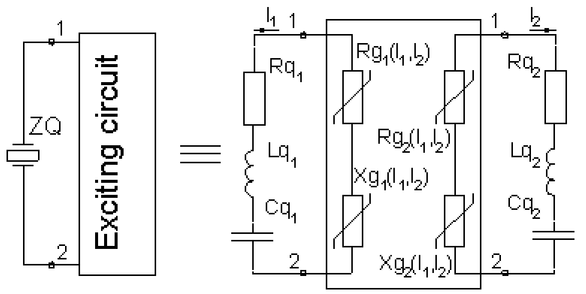


Fig. 3. Dual-mode crystal oscillator macromodel level circuit

The behavior of dual-mode crystal oscillator (Fig. 3) is described by a system of two nonlinear differential equations. Each of them is similarly obtained for the circuit of a Fig. 2. Therefore, analysis is carried out similarly to mention above. For example, the shorted differential equations for amplitude of oscillations of dual-mode crystal oscillator look like:

$$\frac{dI_k}{dt} = \frac{I_k \cdot w_{qk}}{2 \cdot Q_k} \cdot \left(\frac{-Rg_k(I_1, I_2)}{Rq_k} - 1 \right), \text{ k=1..2.} \qquad (7)$$

V. Crystal oscillator simulation procedure

The procedure of crystal oscillator analysis is indicated at Fig. 4. The one-port macromodel of resonator exciting circuit is used at the analysis. Let us consider separate elements of this procedure more closely on an example of Collpitts oscillator circuit.

The source data for this procedure are the oscillator circuit configuration and resonator parameters. At the first stage of simulation the exciting circuit macromodel parameters are calculated. The calculations are provided with help MicroCAP6 program.

Let's remind that in one-port representation the oscillator circuit (concerning resonator pins) is characterized by the averaged on the first harmonic complex impedance. The basis for this is the fact, that for high-Q resonator it is possible to consider a through the resonator current like harmonic current (within one oscillation period). The calculation goal is obtaining of averaged exciting circuit input resistance dependence on amplitude of resonator current.

The analyzable circuit and parameters of models of its components as they look in the MicroCAP6, are indicated in Table 2. The current of the resonator is presented by a harmonic current source, which frequency coincides with resonator resonance frequency (10 MHz).

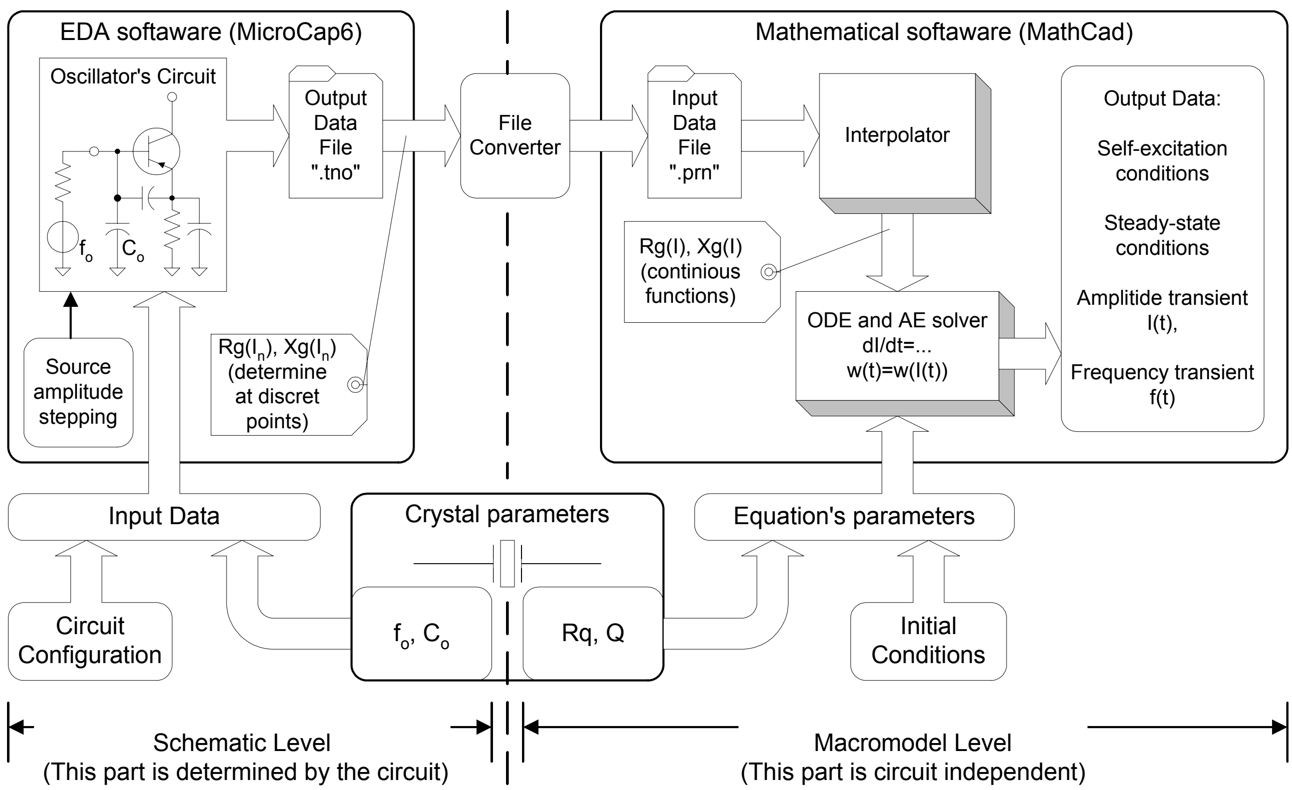
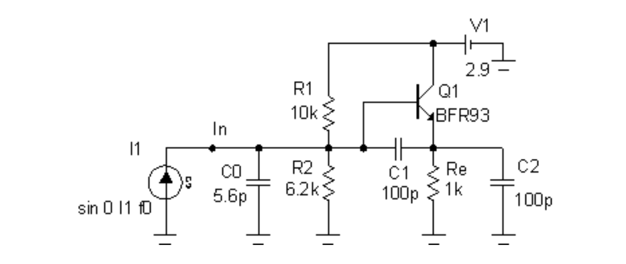


Fig. 4. Schematic diagram of simulation procedure

TABLE 2
Oscillator circuit description in MicroCAP6 program.

Oscillator circuit (drawing window)

Model parameters (text window)
.DEFINE f0 10meg .DEFINE I1 100u .MODEL BFR93 NPN (IS = 4.499E-16 BF = 100 NF = 0.9861 + VAF = 36 IKF = 3 ISE = 7.014E-14 NE = 1.955 BR = 11.95 + NR = 0.985 VAR = 2.8 IKR = 0.02 ISC = 9.97E-16 NC = 1.1 + RB = 10 IRB = 1E-05 RBM = 10 RE = 0.48 RC = 4 XTB = 0 + EG = 1.11 XTI = 3 CJE = 1.72E-12 VJE = 0.88 MJE = 0.37 + CJC = 1.2E-12 VJC = 0.553 MJC = 0.297 XCJC = 0.25 + TF = 1.6E-11 XTF = 500 VTF = 0.9 ITF = 0.9 PTF = 49 + TR = 1E-09 FC = 0.9)

The account is carried out in a “transient” mode. In this mode, it is necessary to execute the following operations sequence:

1. Set the expressions for coordinate axis. On X axe is the current magnitude (in milliamperes) (I1*1k), and on Y axe are the following expressions:

   For account of a real part of exciting circuit resistance $Rg(I)$:
   (T=TMAX)*2*f0*SD((T>=(TMAX-5/f0))*v(In)*sin(2*pi*f0*T))/I1/5

   For account of an imaginary part of exciting circuit resistance $Xg(I)$:
   (T=TMAX)*2*f0*SD((T>=(TMAX-5/f0))*v(In)*cos(2*pi*f0*T))/I1/5

   For extraction of the first harmonics of voltage on an exciting circuit input a Fourier transform is used here. The factor (T>=(TMAX-5/f0)) is necessary to integration performing only for last five periods of oscillations. By this technique using, the initial time interval (when all transients are stabilized) is ignored. Using of small integration interval (one period) is undesirable because of growth of integrations error. The factor (T=TMAX) selects for outcomes obtaining at the end of integration interval.
2. Set the time of analysis (parameter “Time range”) equal to integer number of periods of resonator frequency and has enough large that signal amplitude transient process is over. (5u).
3. Set the parameter "Maximum time step" so there are not less than 100 design points on a period of the input current frequency for adequate accuracy of input impedance evaluation (1n).
4. Click places the waveform in the numeric output file button and set the parameter “number of point” equal to 1.
5. Click the button "Data points" in the view features panel.
6. Choose a type of the initial conditions equal “Leave”.
7. Customize "Stepping" mode: set initial, final values of the signal source amplitude, and step of change. (100u, 5m, 100u)
8. Start transient calculation.

Using of the initial conditions such as “Leave” allows to speed up transient processes when source signal amplitude modify from one value to other and, therefore, allows reducing the time of the analysis (“time range”). However, the diminution of “time range” cans negatively effects on accuracy of exciting circuit resistance calculation at initial value of signal source amplitude. The solution of this problem is the precomputation of the operating point at initial value of signal source amplitude in “operating point only” mode.

The experience of using of MicroCAP6 has shown, that the integration error is reduced, if DC bias on signal source terminals will be close to zero. For DC bias compensation, it is possible to include sequentially with the harmonic current source the compensating voltage source (battery). The value of compensating voltage is defined by the preliminary circuit analysis in DC mode (in our case is 1.09V).

The simulation data of exciting circuit resistance are indicated at Fig. 4. The simulation time (when computer with clock frequency of 800 MHz is used) does not exceed 20 sec.

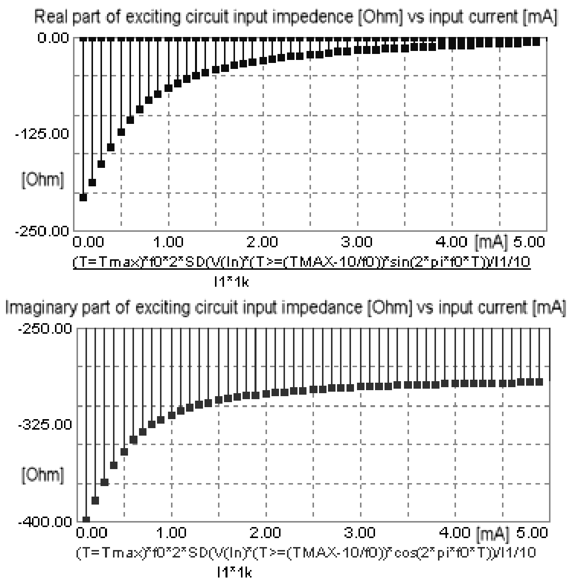


Fig. 4. Outcomes of exciting circuit input resistance calculation in MicroCAP6 program.

The obtained functional dependences (Fig. 4) completely characterize the resonator exciting circuit in one-port representation at crystal resonance frequency. In a matrix format, they are automatically saved in the file with the extension TNO. The given file represents a table macromodel of the resonator exciting circuit. Its will be used for oscillator analysis on macromodel level under the formulas (1)-(6). To use the formulas (1)-(6) it is necessary to translate matrix functional dependences of exciting circuit resistance to continuous one. In this case there is enough effective the cubic spline interpolation using. It is convenient to carry out an interpolation and further calculations in environment of mathematical packages, for example in MathCAD.

Except the useful information, the MicroCAP6 program saves in file with extension TNO miscellaneous supplementary data. For automatic filtering of these data, the following program (on the Q-BASIC language) may be used.

```
filename$ = "llator"
OPEN filename$ + ".tno" FOR INPUT AS #1
OPEN filename$ + ".prn" FOR OUTPUT AS #2
DO
  DO
    LINE INPUT #1, s$
  LOOP WHILE VAL(s$) = 0
  DO
    INPUT #1, I, Rg, Xg
  LOOP WHILE Xg = 0
  PRINT #2, I, Rg, Xg
LOOP WHILE NOT EOF(1)
CLOSE
```

Macromodel of an exciting circuit can be composite. For example, the resistance of an exciting circuit may be not only the one-dimensional function of signal amplitude, but depend on temperature, on component nominal, etc. Calculation of such macromodel parameters will be carried out on the above-stated technique with usage of multiparameter stepping.

The analysis of dual-mode crystal oscillators is performed by similar procedure. The double-frequency harmonic action is specified by two current sources. The frequencies of sources should correspond to the frequencies of resonator modes. The change of oscillations sources amplitude is carried out by means of multiparameter stepping. The example of obtained two-dimensional dependences of exciting circuit resistance on the amplitudes of resonator modes currents are shown at Fig. 5. Here I1 is a C-mode (10 MHz) resonator current, I2 is a B mode (11 MHz) resonator current (dual-mode SC-cut resonator uses).

VI. Computer Analysis of Quartz Oscillator Two-Mode Excitation Stability

One of the primary problems of dual-mode XO design is the maintenance of stability of dual-mode excitation in a wide range of temperatures. Even the small variation of resonator activity frequently leads to the stronger oscillation completely suppresses the weaker one.

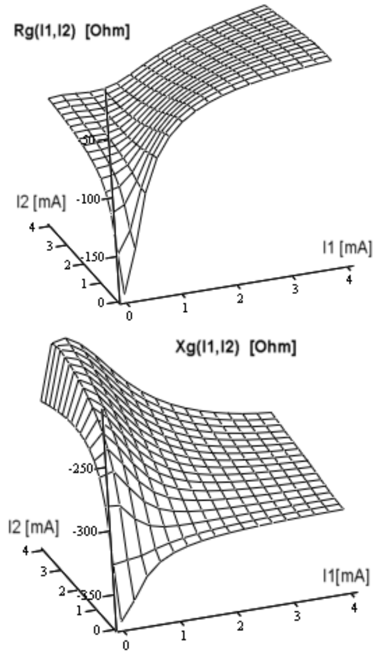


Fig. 5. Two-dimensional dependences of exciting circuit input resistance on the amplitudes of resonator modes currents.

Traditional, the research of stability of XO dual-mode regime is provided by experimental investigation, because of difficulties of dual-mode XO mathematical simulation. The analysis based on the above-stated technique can compensate for this deficiency. The stability condition of XO steady state at dual-mode excitation can be found, having analyzed solutions of (7) on stability using Lyapunov method. Divide one equation on other to remove a variable dt:

$$\frac{dI_1}{dI_2} = \frac{I_1 \cdot \left(Rg_1(I_1,I_2) + Rq_1\right) \cdot Rq_2 \cdot Q_2 \cdot w_{q_1}}{I_2 \cdot \left(Rg_2(I_1,I_2) + Rq_2\right) \cdot Rq_1 \cdot Q_1 \cdot w_{q_2}} . \quad (8)$$

The solution of a system (7) will be critical points of the equation (8), i.e. those points, in which one both numerator and denominator will simultaneously vanish.

Let's consider these critical points:

1. $I_1=I_2=0$. The excitation is absent. For oscillators this point is unstable, and the casual fluctuations force a system to proceed to other statuses of equilibrium.
2. $I_2=0$, $Rg_1(I_1,I_2)+Rq_1=0$ - Single-frequency excitation on the first mode of oscillations.
3. $I_1=0$, $Rg_2(I_1,I_2)+Rq_2=0$. - Single-frequency excitation on the second mode of oscillations.
4. $Rg_1(I_1,I_2)+Rq_1=0$, $Rg_2(I_1,I_2)+Rq_2=0$ - double-frequency excitation. The character of system behavior near to the given point can be anyone and depends on a type of nonlinearities $Rg_1(I_1,I_2)$, $Rg_2(I_1,I_2)$.

Let's enter variables ΔI_1 and ΔI_2, having defined them as a deviation concerning an equilibrium position. From equation (8), using an expansion in Taylor series and cutting the nonlinear members, we will receive the equation of the first approximation:

$$\frac{d\Delta I_1}{d\Delta I_2} = \frac{I_1 \cdot Rq_2 \cdot Q_2 \cdot w_{q1}}{I_2 \cdot Rq_1 \cdot Q_1 \cdot w_{q2}} \cdot \frac{\frac{\partial Rg_1}{\partial I_1} \cdot \Delta I_1 + \frac{\partial Rg_1}{\partial I_2} \cdot \Delta I_2}{\frac{\partial Rg_2}{\partial I_1} \cdot \Delta I_1 + \frac{\partial Rg_2}{\partial I_2} \cdot \Delta I_2}, \quad (9)$$

Hereinafter it is meant, that Rg_1 and Rg_2 is two-dimensional functions of resonator currents amplitudes ($Rg_1(I_1, I_2)$).

In order that the solution in a critical point will be stable it was necessary and enough that the radicals of secular equations composed for equation (9) will be a negative. In result the conditions of stability records in the following form:

$$\frac{\partial Rg_1}{\partial I_1} + \frac{\partial Rg_2}{\partial I_2} > 0, \quad \frac{\partial Rg_1}{\partial I_1} \cdot \frac{\partial Rg_2}{\partial I_2} - \frac{\partial Rg_1}{\partial I_2} \cdot \frac{\partial Rg_2}{\partial I_1} > 0. \quad (10)$$

As it is visible from these conditions, the stability of crystal oscillator dual-mode excitation depends on character of exciting circuit nonlinearity.

The analysis of resonator exciting circuit macromodel parameters with use of given stability conditions allows defining:

1. The area of resonator dynamic resistances on each oscillation mode for which one the double-frequency excitation of given oscillator circuit is possible.
2. The area of deviations of resonator dynamic resistance (for concrete resonator) on each oscillation mode for which one, the double-frequency excitation regime is remains.
3. Factors permitting stimulation or aggravation of dual-mode excitation.

The diagram of stable dual-mode excitation area of the dual-mode Colpitts crystal oscillator [2] in a range of values of resonator modes resistance is showed in the fig. 6.

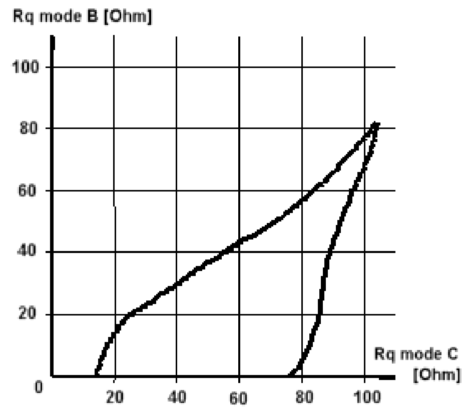


Fig. 6. Stable dual-mode excitation area of the dual-mode Colpitts crystal oscillator [2] in a range of values of resonator modes resistance.

VII. Experimental verification

Any resonator exciting circuit, irrespective of its complexity is reduced to the simple one-port representation characterized by complex impedance. The functional dependence of the equivalent one-port impedance can fully reflect any characteristics of the exciting circuit. For example temperature properties $Rg(I,T)$, effect of supply voltage variation $Rg(I,Vcc)$, etc. Accuracy of oscillator parameters calculation directly depends on accuracy of this functional dependence calculation.

For measurement of the exciting circuit input impedance the circuit, showed in the fig. 7 was constructed.

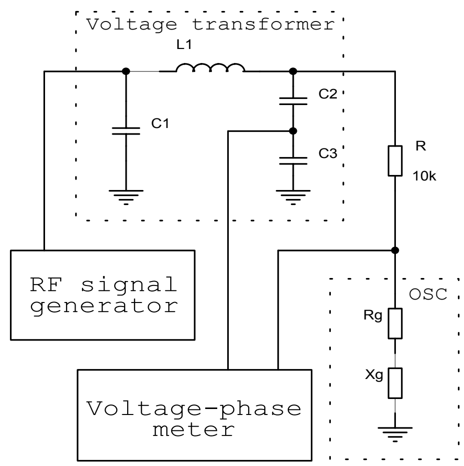


Fig. 7. Input impedance measurement circuit.

For creation of the mode, approximated to a current source, the square-topped tank (units L1, C1, C2, C3), loaded on the high-resistance resistor R was used. The voltage from the tank output feed the first input of the vector voltmeter (voltage-phase meter on fig. 7), and voltage from the input of the excitation circuit feed its second input. The vector voltmeter meters the voltage amplitudes (U1 and U2) and phase shift (φ) of input signals. The capacity divider (units C2, C3) is necessary for limitation of voltage on the first input of vector voltmeter. The measurements are made on the first harmonics of input signals. The real and imaginary part of the linearized input impedance under such circumstances can be defined under the formulas:

$$\begin{cases} Rg = \dfrac{R \cdot k \cdot (\cos\varphi - k)}{1 - 2 \cdot k \cdot \cos\varphi + k^2}, \\ Xg = \dfrac{R \cdot k \cdot \sin\varphi}{1 - 2 \cdot k \cdot \cos\varphi + k^2} \end{cases} \quad (11)$$

where k – ratio of levels of the first harmonic voltages U1 and U2: k = |U1/U2 |. The results of measurements for the real and imaginary parts of the input impedance (for circuit from table 2) are showed in the fig. 8.

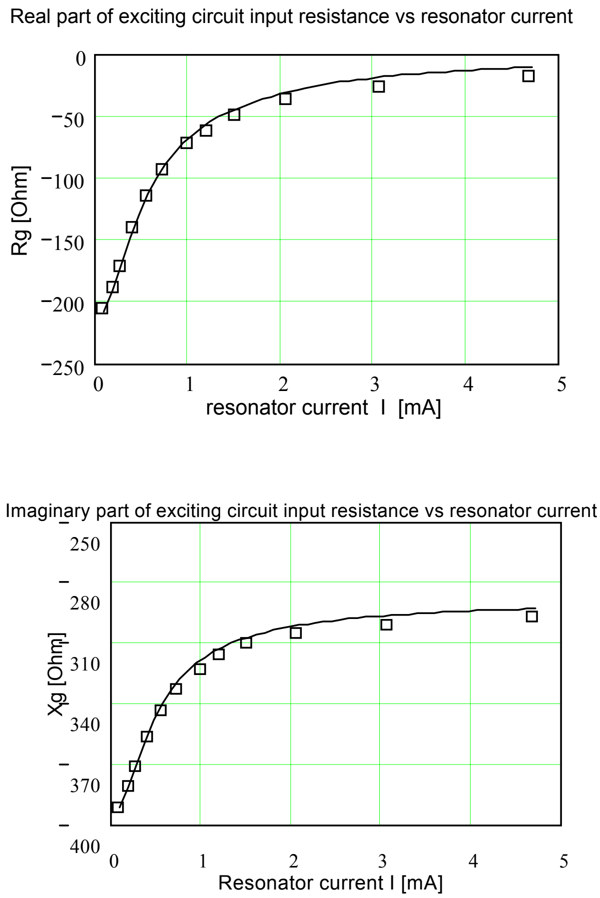


Fig. 8. Simulated and experimental exciting circuit impedance when varying amplitude of resonator current.

In this figure, the solid line is the results of simulation with the help of the program MicroCap6, and small squares - experimental data. It is visible that the results of experiments very well correspond to calculated data.

Accuracy of oscillator parameters calculation depends on one more factor - accuracy of resonator model. The resonator is complex nonlinear system. Resonator replacement by linear model gives high accuracy not at all cases. High accuracy obtained in case of use SC-cut resonator with feebly marked amplitude-frequency effect or in case of small resonator drive level. Use of AT-cut resonator at high drive level requires taking into account resonant frequency variation with excitation power. Nonlinearity of the resonator can be taken into account by approximation of experimental resonator characteristics.

In experiments the resonator with the following parameters was used:

Cut	Overtone number	f, MHz	Q-factor	Rq, Ohm
SC	1	10	294000	9.5

Simulated and experimental relative frequency fluctuation when varying emitter resistance for circuit table 2 is showed in the fig. 9. In fig. 10 it is shown simulated and experimental relative frequency fluctuation when varying supply voltage.

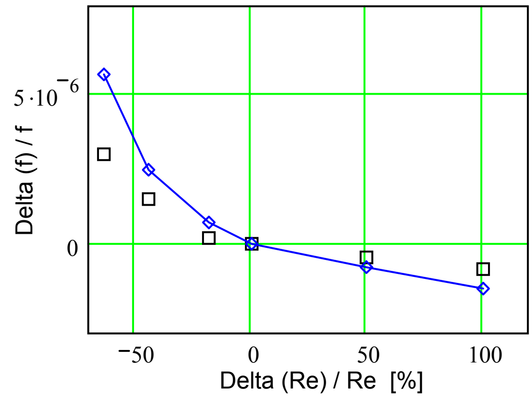


Fig. 9. Simulated and experimental relative frequency fluctuation when varying emitter resistance.

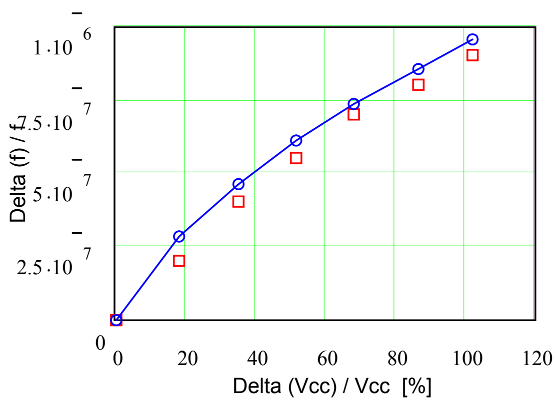


Fig. 10. Simulated and experimental relative frequency fluctuation when varying supply voltage.

The experimental results show that in all cases the value of variation and the order of magnitude of frequency shift have been enough properly predicted.

VII. Conclusion

1. The submission of a crystal oscillator at macromodel level as combinations of resonator and exciting circuit allows applying two-level simulation. Combination of numerical and analytical methods of the analysis at different levels of simulation is raise speed of crystal oscillator simulation more then 200 times.
2. From our point of view, the most rational is the submission of resonator exciting circuit macromodel as a one-port characterized by complex resistance.

3. The technique demonstrated above ensures high simulation accuracy for single-mode oscillators, and for dual-mode regimes gives authentic outcomes, at least, at a qualitative level [2].
4. For further increasing of calculations accuracy it is necessary to take into account the resonator parameters dependences on drive level.

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